the density function is computed, the expected casualty computation can proceed.

In many current display systems the computer functions as a high-speed data processor to accept data from many sources, correlate and collate as necessary, and present significant data to the operators. In the case of a *display system* for range safety operations, the function of the computer is expanded to include real time computation of the numerous parameters required for manual or automatic decision making. In addition, data relating to missile conditions and instrumentation status requires processing to make it more meaningful. The software for computer generated display systems is a separate programing art. Special programing problems arise wherein tradeoffs between computer hardware and display hardware techniques for utilizing data become factors which must be thoroughly studied in the design of a special system.

EROICA Hardware

The information system developed thus far envisions a comprehensive television screen display which provides the RSO a picture, as complete as possible, of integrated decision data elements in real time. A variety of computers and peripheral devices could be used to implement EROICA. Specific computers will not be discussed, but classes of computers with similar characteristics will be evaluated. The two chief factors which relate to the concept are input/output (I/O) features and central processor organization. The following four classes will be considered: 1) single central processor unit; single channel I/O, non-interlaced; 2) single central processor unit; multiple channel I/O with interlace; 3) multiple central processor units; multiple channel I/O with interlace; and 4) multiple computers with channeling; multiple central processor units in main computer; multiple channel I/O with interlace in main computer. Each class can be configured in three versions: unmodified, modified to include large random access memory units, and modified to a hybrid system with an analog computer.

Greater capacity and program flexibility are obtained with increasing complexity, and this capability is bought for increasing cost. In order to utilize best the distinctive features of each class of computer, however, the system programing must be designed on a speed/accuracy compromise and still achieve the basic goals of the EROICA philosophy. An estimation of the achievability of these goals can be made by measuring the time to execute each of the four crucial tasks, described in the previous section, using programs best designed for each class. The total time is then compared to the time available, and this is set by the system requirements independent of the computer in use. Thus minimal configurations are identified.

Table 1 summarizes system requirements that accomplish the EROICA goals in terms of computer design parameters, for minimum programing. An unmodified Class 1 machine is paced at the fastest throughput rate, which implies instruction execution time of about 200 nanosec per instruction. An unmodified Class 2 machine, by fully utilizing the interruptible, interlaced processor, is paced at approximately the 100-msec rate. This implies execution time of about 600 nanosec per instruction and is again unrealizable.

Unmodified Class 3 and 4 machines with three or more central processor units will satisfy EROICA requirements. Again a Class 4 machine in modified configuration provides exceptional flexibility. In this realm far more accurate models of impact prediction, hazard and casualty, and data filters can be implemented.

Table 1 System requirements

Function	Instruction storage	Inputs	Instructions executed per cycle	Throughput rates
filter	100	radar	100	100 msec
impact prediction	200	4 coefficients	160,000	100 msec
impact prediction	200	8 coefficients	320,000	2 sec
hazard prediction	300	50 population centers 4 fragment categories	1,200	100 msec
display	300	computer data	300	33 msec

Tracking Angle Errors due to Frame Misalignment

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TRACKING elevation and/or azimuth errors in the form of residuals or biases that are periodic with respect to azimuth may result from misalignment of the topocentric frame in which elevation and azimuth are measured. This Note presents the derivation of the error formulae resulting from a tilt of the tracking frame about an axis in the horizontal plane. A least-squares procedure is presented by which the misalignment angle, the axis of misalignment and the true bias may be determined from bias or residual data.

Let a topocentric rectangular Cartesian frame be defined with axes directed east (E), north (N), and up (U) as shown in Fig. 1.

Let **e** be the unit vector pointing to a satellite being tracked in the ENU frame. The angles defining the direction of **e** are azimuth (Az) and elevation (El) as shown in Fig. 1. The components of **e** are then given by

$$E = \cos\text{El} \sin\text{Az} \tag{1}$$

$$N = \cos\text{El} \cos\text{Az} \tag{2}$$

$$U = \sin\text{El} \tag{3}$$

Suppose now that the ENU frame is rotated by a small angle θ about an axis in the EN (local horizontal) plane which is directed at an angle ϵ measured counter-clockwise from N. Let the rotated frame have axes E', N', and U' (see Fig. 2).

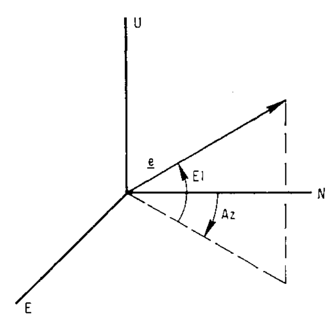

Fig. 1 Topocentric frame.

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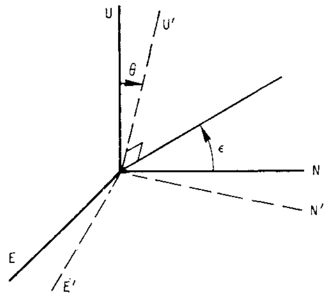

Fig. 2 Displaced frame.

The components of **e** in the primed frame are then

$$E' = \cos\mathrm{El}' \sin\mathrm{Az}' \tag{4}$$

$$N' = \cos\mathrm{El}' \cos\mathrm{Az}' \tag{5}$$

$$U' = \sin\mathrm{El}' \tag{6}$$

where Az′ and El′ are the azimuth and elevation of **e** measured in the primed frame.

It follows that

$$\begin{bmatrix} E' \\ N' \\ U' \end{bmatrix} = \begin{bmatrix} \cos\epsilon & -\sin\epsilon & 0 \\ \sin\epsilon & \cos\epsilon & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \times \begin{bmatrix} \cos\epsilon & \sin\epsilon & 0 \\ -\sin\epsilon & \cos\epsilon & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} E \\ N \\ U \end{bmatrix} \tag{7}$$

Since θ is assumed to be a small angle, Eq. (7) may be linearized with respect to θ and condensed to yield.

$$\begin{bmatrix} E' \\ N' \\ U' \end{bmatrix} \begin{bmatrix} 1 & 0 & -\theta\cos\epsilon \\ 0 & 1 & -\theta\sin\epsilon \\ \theta\cos\epsilon & \theta\sin\epsilon & 1 \end{bmatrix} \begin{bmatrix} E \\ N \\ U \end{bmatrix} \tag{8}$$

Now, combining Eqs. (6) and (8) gives

$$\sin\mathrm{El}' = U' = E\theta\cos\epsilon + N\theta\sin\epsilon + U \tag{9}$$

similarly, Eqs. (4) and (8) yield

$$\sin\mathrm{Az}' = \frac{E'}{\cos\mathrm{El}'} = \frac{E - U\theta\cos\epsilon}{\cos\mathrm{El}'} \tag{10}$$

Let

$$\mathrm{El}' = \mathrm{El} + \delta \tag{11}$$

$$\mathrm{Az}' = \mathrm{Az} + \eta \tag{12}$$

where δ and η are small angles. Substituting Eqs. (1–3, 11, and 12) into Eqs. (9) and (10) results in

$$\sin\mathrm{El}' = \sin\mathrm{El} + \delta\cos\mathrm{El} = \theta[\cos\epsilon\cos\mathrm{El}\sin\mathrm{Az} + \sin\epsilon\cos\mathrm{El}\cos\mathrm{Az}] + \sin\mathrm{El} \tag{13}$$

$$\sin\mathrm{Az}' = \sin\mathrm{Az} + \eta\cos\mathrm{Az} = \frac{\cos\mathrm{El}\sin\mathrm{Az} - \theta\cos\epsilon\sin\mathrm{El}}{\cos\mathrm{El} - \delta\sin\mathrm{El}} \tag{14}$$

Solving Eq. (13) for δ yields

$$\delta = \theta\sin(\mathrm{Az} + \epsilon) \tag{15}$$

Now, combining Eqs. (14) and (15) yields

$$\eta = -\theta\tan\mathrm{El}\cos(\mathrm{Az} + \epsilon) \tag{16}$$

The quantities δ and η represent the respective errors in elevation and azimuth resulting from taking measurements in the primed frame. Note from Eq. (15) that the elevation error δ is independent of the elevation being measured.

If elevation is measured with respect to the vertical (U) axis only (instead of with respect to the E-N plane), and if the U axis and the E-N plane are assumed to be separately misaligned, then it can be shown that elevation and azimuth errors may occur independently. Let the U axis be tilted by θ_1 (resulting in U') about an axis in the E-N plane rotated ϵ_1 counter-clockwise from N, and let the E-N plane be tilted by θ_2 (resulting in axes E' and N') about an axis rotated ϵ_2 from N. Then, if elevation measurements are taken with respect to U' only, the following error formulae result

$$\delta = \theta_1\sin(\mathrm{Az} + \epsilon_1) \tag{17}$$

$$\eta = -\theta_2\tan\mathrm{El}\cos(\mathrm{Az} + \epsilon_2) \tag{18}$$

Therefore, if elevation and/or azimuth residuals from a tracked satellite exhibit a periodic behavior with azimuth, frame misalignment may be suspected. The detection of such errors may be complicated by relatively short azimuth and/or low elevation sweeps during a satellite pass. In such cases azimuth and/or elevation errors might be interpreted as measurement biases and be corrected out (in a least squares sense) with constant bias values. Therefore, it might require reasonably high-elevation angles and an azimuth range sufficient to reveal periodic behavior of residuals in order to detect frame misalignment errors from tracking data.

For example, suppose a "stationary" (24 hr circular equatorial) satellite were being tracked in a frame whose vertical axis were misaligned. Then, as long as the satellite is stationary, a constant elevation bias will produce good tracking data. However, if the satellite starts to "drift" in longitude, the elevation bias (computed, for example, daily) will change with longitudinal position and consequently with azimuth. Although, a short drift arc and measurement noise might produce elevation biases which do not appear periodic with azimuth, the data may still be used to determine the misalignment by a least-squares fit.

Assume in the preceding example that n values of elevation bias B are computed when the satellite is in drift mode. Let K be the true elevation bias and δ the elevation error due to misalignment of the vertical. Then with the use of Eq. (15), the ith measured bias may be represented as

$$B_i = K + \theta\sin(\mathrm{Az}_i + \epsilon) \tag{19}$$

where Az_i is the average azimuth over the bias determination period. The residual r_i of the bias B_i is then

$$r_i = B_i - K - \theta\sin(\mathrm{Az}_i + \epsilon) \tag{20}$$

A least-squares formulation for the determination of θ, K, and ϵ yields

$$\theta = \frac{\Sigma B_i\sin(\mathrm{Az}_i + \epsilon) - (1/n)\Sigma B_i\Sigma\sin(\mathrm{Az}_i + \epsilon)}{\Sigma\sin^2(\mathrm{Az}_i + \epsilon) - (1/n)[\Sigma\sin(\mathrm{Az}_i + \epsilon)]^2} \tag{21}$$

$$K = \frac{\Sigma B_i\Sigma\sin^2(\mathrm{Az}_i + \epsilon) - \Sigma\sin(\mathrm{Az}_i + \epsilon)\Sigma B_i\sin(\mathrm{Az}_i + \epsilon)}{n\Sigma\sin^2(\mathrm{Az}_i + \epsilon) - [\Sigma\sin(\mathrm{Az}_i + \epsilon)]^2} \tag{22}$$

$$\theta\Sigma\sin 2(\mathrm{Az}_i + \epsilon) - 2\Sigma B_i\cos(\mathrm{Az}_i + \epsilon) + 2K\Sigma\cos(\mathrm{Az}_i + \epsilon) = 0 \tag{23}$$

where the summation range of i is from 0 to n.

Note that ϵ does not appear explicitly in the preceding three equations. Therefore, in order to determine ϵ, one should guess an initial value for ϵ, compute θ and K from Eqs. (21) and (22) and determine if Eq. (23) is satisfied. If Eq. (23) is not satisfied by the trial value of ϵ, then the procedure should be iterated with better estimates of ϵ possibly obtained from a Newton-Raphson formulas until Eq. (23) is satisfied. The parameters K, θ, and ϵ obtained by this method are respectively the least squares estimates of the true elevation bias, the vertical misalignment, and the position of the misalignment axis.

A similar procedure may be followed to determine the tilt of the horizontal plane if this condition is suspected or if periodic azimuth biases are found. The above procedure is considerably simplified if the true bias K is known or is determined independently. If K is known, its value may be subtracted from the B_i in Eq. (19). The conditions for a least-squares data fit then become

$$\theta = \frac{\Sigma B_i \sin(\mathrm{Az}_i + \epsilon)}{\Sigma \sin^2(\mathrm{Az}_i + \epsilon)} \quad (24)$$

$$F \triangleq \Sigma \sin 2(\mathrm{Az}_i + \epsilon)\Sigma B_i \sin(\mathrm{Az}_i + \epsilon) - 2\Sigma \sin 2(\mathrm{Az}_i + \epsilon)\Sigma B_i \cos(\mathrm{Az}_i + \epsilon) = 0 \quad (25)$$

From Eq. (25)

$$F' \triangleq \frac{\partial F}{\partial \epsilon} = 2\Sigma \cos^2(\mathrm{Az}_i + \epsilon)\Sigma B_i \sin(\mathrm{Az}_i + \epsilon) - \Sigma \sin 2(\mathrm{Az}_i + \epsilon)\Sigma B_i \cos(\mathrm{Az}_i + \epsilon) \quad (26)$$

The Newton-Raphson condition for determining ϵ may then be given as

$$\epsilon_{j+1} = \epsilon_j - F(\epsilon_j)/F'(\epsilon_j) \quad (27)$$

where ϵ_0 is the initial guess and ϵ_j as the jth approximation to ϵ.

On Methods for Determining the Composition of Pyrolysis Products from Ablative Composites

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Nomenclature

C_p	= heat capacity at constant pressure
$\Delta H_f, \Delta H_C, \Delta H_{\mathrm{pyr}}$	= heats of formation, combustion and pyrolysis, respectively
K, M	= total number of products, and reactants, respectively
T	= temperature
x	= mass fraction

Subscripts

i,p,r	= ith species, products and reactants, respectively
pyr	= pyrolysis

Introduction

KNOWLEDGE of the composition of the pyrolysis products from the decomposition zone of a charring ablator is needed in the analysis of the heat-transfer processes taking place in the char zone and in the boundary layer. Direct measurement of the composition of the pyrolysis products is desirable, but there are difficulties in obtaining representative samples.[1-8] Consequently, most of the data reported in the literature present only those components that were gases near room temperature, which nominally account for only about 50% by weight of the total material pyrolyzed. Other methods are needed, and three alternative methods are presented and used for comparison with the compositions obtained from the chemical analysis of pyrolysis products from ablative composites.

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Methods for Estimating Pyrolysis Gas Compositions

The total of four methods that can be used together to determine the composition of the pyrolysis products from ablative composites are discussed in the following subsections.

Chemical analysis of the pyrolysis gases

Sykes[2,3] degraded ablative composites in a furnace and injected the hot pyrolysis products directly into a gas chromatograph. However, even with his careful procedure, there was a certain amount of condensation of high-molecular-weight species in the heated line between the furnace and the chromatograph which remained unidentified. The method has, however, reduced the total amount of unidentified material pyrolyzed from the 50% level obtained with the more conventional methods[1] to 17% (by weight), and the species and their concentrations were more precisely determined. Therefore, results from Sykes' technique will form the basis for selecting a pyrolysis product composition used in the following method. The last two methods were used to make adjustments to the analytically determined composition, especially with regard to the species which would logically make up the unidentified portion of the pyrolysis products.

Comparison of measured and calculated heats of pyrolysis

The heat of pyrolysis, ΔH_{pyr}, which converts the virgin plastic composite ("reactants") to pyrolysis gases plus char (in total, the "products") can be experimentally measured by differential thermal analysis (DTA). Also, it can be calculated knowing the heats of formation and specific heat $C_p(T)$ of the products and reactants:

$$\Delta H_{\mathrm{pyr}} = \sum_{i=1}^{M}\left[x_{p,i}\,\Delta H_{f,pi} + \int_{25°\mathrm{C}}^{T_p} x_{p,i}C_{p,pi}dT\right] - \sum_{j=1}^{M}\left[x_{r,j}\Delta H_{f,rj} + \int_{25°\mathrm{C}}^{T_r} x_{r,j}C_{p,rj}dT\right] \quad (1)$$

For nylon-phenolic resin composites, the temperature T_r, where degradation starts, is approximately 250°C, and the temperature where the degradation ends is approximately 1000°C. Pyrolysis products are generated over this temperature range from 250 to 1000°C, and T_p is the average temperature which gives the correct energy associated with the pyrolysis products. It was determined to be 700°C as a weighted average based on the mass loss rate. The heats of formation of nylon and phenolic resin, $\Delta H_{f,rj}$, were calculated from experimentally determined heats of combustion, ΔH_C, by the procedure reported in Ref. 9. Then the ΔH_{pyr} can be computed if the pyrolysis gas composition x_{pi} is known since $\Delta H_{f,pi}$ is tabulated in standard references e.g., Ref. 10.

If the value of ΔH_{pyr} calculated by the foregoing method does not agree within 10% of the value measured by DTA, the composition proposed is either in error or is incomplete. A recommended way to obtain an accurate composition is to inspect all of the available data as reported by various investigators and select those species to be present which are reported by a majority of the investigations. If the composition of various fractions are given, weigh each species composition according to the size of each fraction. Then superimpose these results and construct an over-all species composition listing. Average the values of species that appear within reasonable agreement, and use the species that are reported in a minority of the papers to make minor adjustments to the composition.